Stochastic quantization of Born-Infeld field

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Abstract

We [1] stochastically quantize the Born-Infeld field which can hardly be dealtwith by means of the standard canonical and/or path-integral quantization methods. [2] We set a hypothetical Langevin equation in order to quantize the Born-Infeld field, following the basic idea of stochastic quantization method. Numerically solving this nonlinear Langevin equation, we obtain a sort of "particle mass" associated with the gauge-invariant Born-Infeld field as a function of the so-called universal length.

1 Introduction

Many years ago Born and Infeld [1] presented a nonlinear electromagnetic field with a non-polynomial action including the so-called *universal length*. One of the most important characteristics of the Born-Infeld field is found in its static solution which has no divergence of static self-energy. Many physicists expected that this might be an example of divergence-free field theory. However, no one could succeed to quantize the field, by means of the standard canonical quantization method, because of the complicated nonlinearity. Even the path-integral quantization can hardly be applied to this field, because we cannot easily manipulate such a non-polynomial action. We have to invent a new quantization method if we want to quantize the Born-Infeld field.

About ten years ago, Parisi and Wu [2] proposed a new quantization method, called *stochastic quantization*, by introducing a hypothetical stochastic process with respect to a new (fictitious) time, say t, other than ordinary time, say x_0. The stochastic process is so designed as to yield quantum mechanics in the infinite t-limit (thermal equilibrium limit). This theory starts from a

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[2] This is a revised version of which original non-electronic one was published in 1995 by RISE in Waseda Univ.

hypothetical Langevin equation for the stochastic process, given by adding the fictitious-time derivative and the random source to the classical equation of motion. Remember that the stochastic quantization can be formulated only on the basis of classical field equation, without resorting to canonical formalism.

2 Brief review of Born-Infeld field

The ordinary electromagnetic field is described by the following Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) \ , \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The corresponding action is given by

$$\mathcal{S} = \int d^4x \mathcal{L} \ . \tag{2}$$

Here we have followed the usual notation. Note that we keep the Minkowski metric.

In the case of spherically symmetric static electric field, we can put

$$\mathbf{E} = -\nabla A_0(r), \qquad \mathbf{H} = 0 \ , \tag{3}$$

and obtain

$$A_0 = \frac{e}{r} \tag{4}$$

for a point charge e. That this field becomes ∞ for $r \to 0$ is a well-known fact.

Let us introduce the action functional of the Born-Infeld field [1] :

$$\mathcal{S}_B = \int d^4x \left[-b^2\sqrt{1 - \frac{2}{b^2}\mathcal{L}} + b^2 \right] \ , \tag{5}$$

where $1/\sqrt{b}$ is a sort of universal constant called *universal length*, and has the dimension of length in natural unit $\hbar = c = 1$. We can easily see that $\mathcal{S}_B \to \mathcal{S}$, that is, the Born-Infeld field will become the ordinary electromagnetic field as b tends to ∞.

As is well known, all physical quantities can be written only in terms of the dimension of length in natural unit $\hbar = c = 1$. Many years ago Heisenberg anticipated that we could formulate a finite field theory, free from field-theoretical divergences, if we could bring a sort of *universal length* into physics in an appropriate way. Following his idea, Born and Infeld [1] proposed to use the above field given by action (5). Unfortunately, however, the Heisenberg's anticipation was not accomplished yet even now.

For the spherically symmetric static field $A_0(r)$ (3), the Born-Infeld action (5) yields the following equation

$$\frac{\partial}{\partial r} \left[r^2 \frac{\frac{\partial}{\partial r} A_0(r)}{\sqrt{1 - \frac{1}{b^2}(\frac{\partial}{\partial r} A_0(r))^2}} \right] = 0 \; , \tag{6}$$

whose solution is given by

$$A_0 = \frac{e}{r_0} \int_{r/r_0}^{\infty} d\xi \frac{1}{\sqrt{1 + \xi^4}} \; \rightarrow \; \begin{cases} (e/r) & \text{for } r \gg r_0 \; , \\ 1.8541 \cdot (e/r_0) & \text{for } r \to 0 \; , \end{cases} \tag{7}$$

where $r_0 = \sqrt{|e|/b}$. We surely realize that the Born-Infeld field has finite static self-energy. Of course, the self-energy goes back to the original infinity as $b \to \infty$.

We are not interested in static field but in wave field propagating to remote places. The Euler-Lagrange equation of $\mathcal{S}_B$ is generally written as

$$\frac{\delta \mathcal{S}_\mathcal{B}}{\delta A_\nu(x)} = \partial_\mu \left[\frac{F^{\mu\nu}}{\sqrt{1 + \frac{1}{2b^2} F^2}} \right] = \partial_\mu \left[\frac{F^{\mu\nu}}{\chi} \right] = 0 \; , \tag{8}$$

or

$$\partial_\mu F^{\mu\nu} = (\partial_\mu \ln \chi) F^{\mu\nu} \; , \tag{9}$$

where $F^2 = F_{\mu\nu} F^{\mu\nu}$ and $\chi = \sqrt{1 + \frac{1}{2b^2} F^2}$.

The right-hand side of this equation is proportional to $1/b^2$, and can be expanded in a power series of $1/b^2$. Needless to say, its unperturbed one (for $1/b = 0$) is nothing other than the free Maxwell equation

$$\partial_\mu F^{\mu\nu} = 0 \; , \tag{10}$$

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which allows us to use the wave gauge given by

$$A_0 = 0 \ , \quad \nabla \cdot \mathbf{A} = 0 \ . \tag{11}$$

Consequently, one may naively expect to have the perturbative theory based on (9) and (11). In this case, however, we can hardly develop this kind of the perturbative approach to the quantized Born-Infeld field, because the interaction part includes higher powers of derivative terms. This is the reason why we attempt to develop an unperturbative approach to the quantized Born-Infeld field by means of *stochastic quantization* [2] in the present paper.

3   Stochastic quantization

As is well-known, it is convenient to use the Euclidean action $\mathcal{S}_E$ derived by the Wick rotation $(x_0 \rightarrow -ix_0)$ from the original Minkowski action, for the purpose of carrying out the Parisi-Wu stochastic quantization. In order to perform *stochastic quantization* of the Born-Infeld field A_μ, we have to introduce the additional dependence on fictitious-time t (other than ordinary-time x_0) into the field quantities and then to set the basic Langevin equation

$$\frac{\partial}{\partial t} A_\mu(x,t) = -\frac{\delta \mathcal{S}_E}{\delta A_\mu}\big|_{A=A(x,t)} + \eta_\mu(x,t) \tag{12}$$

for a hypothetical stochastic process of $A_\mu(x,t)$ with respect to t [2]. Here $\eta_\mu(x,t)$ is the Gaussian white-noise field subject to

$$< \eta_\mu(x,t) > = 0 \ , \tag{13}$$
$$< \eta_\mu(x,t)\eta_\nu(x',t') > = 2\delta_{\mu\nu}\delta(x-x')\delta(t-t') \tag{14}$$

for statistical ensemble averages, where we have put $\hbar = 1$ for simplicity.

According to the prescription of stochastic quantization [2], we can derive the field-theoretical propagaters through the well-known formula

$$
\begin{aligned}
&D_{\mu\nu}^A(x,x') \\
&= \lim_{t\to\infty} \left[< A_\mu(x,t)A_\nu(x',t) > - < A_\mu(x,t) >< A_\nu(x',t) > \right] \ ,
\end{aligned} \tag{15}
$$

where $A_\mu(x,t)$ as a function of η_μ is to be obtained by solving (12).

Let us decompose A_μ and η_μ into their longitudinal and transverse components, A_μ^L and A_μ^T, and η_μ^L and η_μ^T, given by

$$A_\mu^L = \frac{1}{\Box}\partial_\mu\partial_\nu A_\nu \tag{16}$$

$$A_\mu^T = (\delta_{\mu\nu} - \frac{1}{\Box}\partial_\mu\partial_\nu)A_\nu \; ; \;\; \partial_\mu A_\mu^T = 0 \; , \tag{17}$$

and similar ones for η's. Therefore, we can decompose the basic Langevin equation (12) as follows;

$$\frac{\partial}{\partial t}A_\mu^L(x,t) = 0 + \eta_\mu^L(x,t) \; , \tag{18}$$

$$\frac{\partial}{\partial t}A_\mu^T(x,t) = -\frac{\delta\mathcal{S}_E}{\delta A_\mu^T}|_{A^T=A^T(x,t)} + \eta_\mu^T(x,t) \; . \tag{19}$$

The absence of drift force in (18) is an important reflection of the gauge invariance that $\mathcal{S}_E$ does not depend on A_μ^L. Consequently, the longitudinal component $A_\mu^L(x,t)$ makes a random walk around its initial value $A_\mu^L(x,0) = \frac{1}{\Box}\partial_\mu\phi(x)$, $\phi(x)$ being a scalar field. As was discussed in detail in the case of non-Abelian gauge field [3], we must introduce *gauge parameter* α by taking average of ϕ over random fluctuations around zero $(\overline{\phi(x)\phi(x')} = \alpha\delta(x - x'))$. Thus we obtain

$$D_{\mu\nu}^{A^L}(x,x') = \frac{1}{\Box}\partial_\mu\partial_\nu(\alpha\frac{1}{\Box} + 2t)\delta(x - x') \; . \tag{20}$$

Of course, we know that the longitudinal components never appear in gauge-invariant (physical) quantities.

We should also notice that the transverse components and their propagaters are completely decoupled with the longitudinal ones in the present case. This is an important point quite different from the non-Abelian gauge field case. Thus we can safely discard A_μ^L, and use the wave gauge (11), even in the present case, for the purpose of deriving propagaters of the transverse components.

Considering wave propagation along the $z = x_3$-axis, we put

$$A_0 = 0 \; , \qquad A_3 = 0 \; , \tag{21}$$
$$A_1 = A_1(x_0, x_3) \; , \qquad A_2 = A_2(x_0, x_3) \; , \tag{22}$$

and

$$\eta_0 = 0 \; , \qquad \eta_3 = 0 \; , \tag{23}$$
$$\eta_1 = \eta_1(x_0, x_3) \; , \qquad \eta_2 = \eta_2(x_0, x_3) \; . \tag{24}$$

Note that A_μ and η_μ have no longitudinal components.

In this case, the Euclid action becomes

$$\mathcal{S}_E = \int dx_0 dx_3 \left[b^2 \sqrt{1 + \frac{1}{b^2} F_E^2} - b^2 \right] , \tag{25}$$

where $F_E^2 = (\partial_0 A_1)^2 + (\partial_3 A_2)^2 + (\partial_0 A_2)^2 + (\partial_3 A_1)^2$. The corresponding classical field equation is given by

$$\frac{\delta S_E}{\delta A_i(x)} = -\partial_0 \left[\frac{\partial_0 A_i}{\sqrt{1 + \frac{1}{b^2} F_E^2}} \right] - \partial_3 \left[\frac{\partial_3 A_i}{\sqrt{1 + \frac{1}{b^2} F_E^2}} \right] = 0, \quad (i = 1, 2) . \tag{26}$$

Note that the dimensions of the field and b are different from the original ones: $[A_i] = [L^0]$ and $[b^{-1}] = [L]$, $[L]$ standing for the dimension of length.

Based on the classical equation, we can set up the basic Langevin equation for stochastic quantization of the Born-Infeld field as follows;

$$\frac{\partial}{\partial t} A_i(x_0, x_3, t) = \partial_0 \left[\frac{\partial_0 A_i}{\sqrt{1 + \frac{1}{b^2} F_E^2}} \right] + \partial_3 \left[\frac{\partial_3 A_i}{\sqrt{1 + \frac{1}{b^2} F_E^2}} \right] + \eta_i(x_0, x_3, t) ,$$

$$(i = 1, 2) , \tag{27}$$

where the fluctuating source-field η_i should have the statistical properties

$$< \eta_i(x_0, x_3, t) >_\eta = 0 , \tag{28}$$
$$< \eta_i(x_0, x_3, t) \eta_j(x_0', x_3', t') >_\eta = 2\delta_{ij}\delta(x_0 - x_0')\delta(x_3 - x_3')\delta(t - t') . \tag{29}$$

Consequently, we have to obtain A_i as a function (or functional) of η_i, by solving (27), and then to calculate expectation values of physical quantities, by making use of (28) and (29). For example, the field-theoretical propagater of A_i is given by the formula

$$\Delta_{ij}^A(x_0 - x_0', x_3 - x_3') \equiv \lim_{t \to \infty} [< A_i(x_0, x_3, t) A_j(x_0', x_3', t) >$$
$$- < A_i(x_0, x_3, t) >< A_j(x_0', x_3', t) >]. \tag{30}$$

For conventional fields, we can extract real information about the particle mass, $\mathcal{M}$ and $\mathcal{M}'$, associated with the field (or the first energy gap) from the asymptotic formulas

$$\Delta_{ii}^A(0, x_3) \overset{|x_3| \to \infty}{\longrightarrow} \text{const.} \exp[-\mathcal{M}|x_3|] , \tag{31}$$

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$$\Delta_{ii}^{A}(x_0, 0) \overset{|x_0| \to \infty}{\longrightarrow} \text{const. exp}[-\mathcal{M}'|x_0|] \tag{32}$$

(i: no summation), in which we can put $\mathcal{M} = \mathcal{M}'$ for the Euclidean symmetry in space-time. Unfortunately in the Born-Infeld case, however, we have no reliable theory to justify the procedure (31) and/or (32) to give mass. Despite of this situation, we intend to follow the conventional approach to the "particle mass" associated with the (transverse) Born-Infeld field, based on (31) and/or (32).

4 Numerical simulation and particle mass

Needless to say, we know that it is very difficult to solve (27) analytically, so that we are inevitably enforced to deal with it by means of numerical simulation. For this purpose, we first discretize the Langevin equation (27) on an $N \times N$ lattice with spacings Δx_0 and Δx_3 (along time and space directions, respectively). Denoting the Born-Infeld field on the (k, l)-th lattice point by $A_{i;k,l}(t)$, where i stands for the i-th component of the field and k, l for the ordinary time and spatial position, then we write down the discretized Langevin equation as

$$\frac{A_{i;k,l}(t + \Delta t) - A_{i;k,l}(t)}{\Delta t}$$
$$= \frac{G_{i;k+1,l}(t) - G_{i;k,l}(t)}{\Delta x_0} + \frac{H_{i;k,l+1}(t) - H_{i;k,l}(t)}{\Delta x_3} + \sqrt{\frac{2}{\Delta x_0 \Delta x_3 \Delta t}} N_{i;k,l}(t) , \tag{33}$$

where

$$G_{i;k,l}(t) = \frac{\frac{1}{\Delta x_0}(A_{i;k,l}(t) - A_{i;k-1,l}(t))}{\sqrt{1 + F_E^2}} , \tag{34}$$

$$H_{i;k,l}(t) = \frac{\frac{1}{\Delta x_3}(A_{i;k,l}(t) - A_{i;k,l-1}(t))}{\sqrt{1 + F_E^2}} \tag{35}$$

with

$$F_E^2 = \frac{1}{b^2} \left\{ \left[\frac{A_{1;k,l}(t) - A_{1;k-1,l}(t)}{\Delta x_0} \right]^2 + \left[\frac{A_{2;k,l}(t) - A_{2;k-1,l}(t)}{\Delta x_0} \right]^2 \right.$$
$$\left. + \left[\frac{A_{1;k,l}(t) - A_{1;k,l-1}(t)}{\Delta x_3} \right]^2 + \left[\frac{A_{2;k,l}(t) - A_{2;k,l-1}(t)}{\Delta x_3} \right]^2 \right\} \tag{36}$$

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for drift terms, and

$$< N_{i;k,l}(t) >_N = 0, \qquad < N_{i;k,l}(t)N_{i;k',l'}(t') >_N = \delta_{ij}\delta_{kk'}\delta_{ll'}\delta_{tt'} , \qquad (37)$$
$$(i,j = 1,2), \qquad (l,k = 1,\ldots,N)$$

for noise terms.

Here let us introduce a scale unit a, which has dimension of length, and put relevant quantities in the following way:

$$\Delta x_0 = \Delta\tilde{x}_0 a, \ \Delta x_3 = \Delta\tilde{x}_3 a, \ \Delta t = \Delta\tilde{t}a^2, \ b = \tilde{b}a^{-1},$$
$$A_{i;k,l}(t) = \tilde{A}_{i;k,l}(\tilde{t}), \ G_{i;k,l}(t) = \tilde{G}_{i;k,l}(\tilde{t})a^{-1},$$
$$H_{i;k,l}(t) = \tilde{H}_{i;k,l}(\tilde{t})a^{-1}, \ N_{i;k,l}(t) = \tilde{N}_{i;k,l}(\tilde{t}),$$
$$\mathcal{M} = \tilde{\mathcal{M}}a^{-1}, \qquad (38)$$

Note that all quantities with tilde are dimensionless. Thus, the equations (33) and (37) are rewritten as

$$\frac{\tilde{A}_{i;k,l}(\tilde{t} + \Delta\tilde{t}) - \tilde{A}_{i;k,l}(\tilde{t})}{\Delta\tilde{t}}$$
$$= \frac{\tilde{G}_{i;k+1,l}(\tilde{t}) - \tilde{G}_{i;k,l}(\tilde{t})}{\Delta\tilde{x}_0} + \frac{\tilde{H}_{i;k,l+1}(\tilde{t}) - \tilde{H}_{i;k,l}(\tilde{t})}{\Delta\tilde{x}_3} + \sqrt{\frac{2}{\Delta\tilde{x}_0\Delta\tilde{x}_3\Delta\tilde{t}}}\tilde{N}_{i;k,l}(\tilde{t}) ,$$
$$(39)$$
$$< \tilde{N}_{i;k,l}(\tilde{t}) >_{\tilde{N}} = 0, \qquad < \tilde{N}_{i;k,l}(\tilde{t})\tilde{N}_{i;k',l'}(\tilde{t}') >_{\tilde{N}} = \delta_{ij}\delta_{kk'}\delta_{ll'}\delta_{\tilde{t}\tilde{t}'} . \qquad (40)$$

$\tilde{G}_{i;k,l}$ and $\tilde{H}_{i;k,l}$ include the dimensionless $\tilde{F}_E^2$ in the same way as $G_{i;k,l}$ and $H_{i;k,l}$ depend on F_E^2. Note that only $\tilde{F}_E^2$ contains the Born-Infeld parameter $\tilde{b}$.

In the conventional field theory, this scale unit a can be determined by making use of the renormalization group theory. In the present case, however, we have no reliable theory to determine the scale unit, and then we shall inevitably calculate all quantities (in particular, the "particle mass") on an arbitrary scale. Only for the sake of simplicity, let us put $a = 1$, in order to go on our procedure. For a while from now on, we suppress those tilders which are put on the quantities. Therefore, the "particle mass", $\mathcal{M}$, will be given as a dimensionless quantity in this scheme.

In order to solve numerically the above equation and obtain the field-theoretical propagater $\Delta_{ij}(x_0, x_3)$ on the above lattice, we should introduce the periodic boundary condition given by

$$A_{i;k,l+2l_c} = A_{i;k,l} , \ A_{i;k+2k_c,l} = A_{i;k,l} , \qquad (41)$$

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Fig. 1. Field-theoretical propagaters for several values of b^{-1}.

where l_c, k_c and $2l_c, 2k_c$ stand for the lattice center and the period, respectively. Practically, we have used the Langevin-source method (for example, see [2]), in which we have performed 5.4×10^6 iterations for a lattice of 20×20 sites with $\Delta t = 0.01$ and $l_c = k_c = 10$ (to realize thermal equilibrium), and then use the subsequent 2.0×10^5 iterations to calculate the field-theoretical propagater.

Figure 1 shows our numerical results of the field-theoretical propagaters $\Delta_{11}(0, x_3)$ for $b^{-1} = 10, 20, 30, 40$ and 50, together with curves given by the asymptotic formula

$$\Delta_{ii}(X, 0) = \Delta_{ii}(0, X) = C \frac{\cosh \mathcal{M} |X - x_c|}{\cosh \mathcal{M} |x_c|} \quad (i : \text{no summation}), \qquad (42)$$

where x_c stands for the center of lattice. Also we numerically have got similar field-theoretical propagaters for other directions and/or components. Equation (42) is the substitute of (31) and/or (32) under the boundary condition (41). We have estimated the "particle mass", $\mathcal{M}$, associated with the Born-Infeld field, by making use of χ^2-fitting based on (42). As shown in Table 1, our results are the following: $\mathcal{M} = 0.0415, 0.0835, 0.1260, 0.1693, 0.2162$, correspondingly to $b^{-1} = 10, 20, 30, 40, 50$, where χ^2 is 4.69×10^{-4}. We have estimated the statistical fluctuations for the fictitious time as accuracies in Table 1. Here we have put $C = \Delta_{ii}(0, 0) = < A_i^2 >$ (i: no summation). Note that C is independent of i due to the space-time uniformity, and that C is gauge-invariant.

Rigorously speaking from the point of view of (38), we can only assert that the above $\mathcal{M}$ is proportional to the "particle mass" associated with the (transverse) Born-Infeld field. We should repeat that we have no renormalization group theory to give the scaling formula in the case of Born-Infeld field. Re-

Table 1
The "particle mass", $\mathcal{M}$, associated with the Born-Infeld field

b^{-1}	10	20	30	40	50
$\mathcal{M}$	0.0415	0.0835	0.1260	0.1693	0.2162
Accuracy	±0.0062	±0.0021	±0.0017	±0.0017	±0.0015

Fig. 2. "Particle mass", $\mathcal{M}$, as a function of b^{-1}.

Fig. 3. "Particle mass", $\mathcal{M}'$, as a function of b^{-1}.

member that the problem is still open to questions. In this paper, however, we are talking about the "particle mass" by $\mathcal{M}$ which is given by (42).

Figures 2 and 3 plot the "particle mass", given by $\mathcal{M}$ and $\mathcal{M}'$, as a function of b^{-1}. Observe that $\mathcal{M} = \mathcal{M}'$. It seems that the "particle mass" is proportional to b^{-1}, but unfortunately, we do not know what kind of physical implications this fact suggests. Another important point should be that the "particle mass" seems vanishing, in the case of $b^{-1} = 0$, as expected from the fact that the Born-Infeld field must go back to the free Maxwell field in this limit. All results are presented in Table 1.

Table 1 also tells us that the (dimensionless) "particle mass" on this scale (with $a = 1$) distributes over a very small region.

Table 1 or Figures 2 and 3 can be fitted well by a single formula given by

$$\mathcal{M} = \gamma \frac{1}{b} , \qquad \gamma = 0.00426 . \tag{43}$$

This equation is rewritten in terms of $\mathcal{M}$ and b^{-1} having dimension as

$$\mathcal{M} = (\frac{\gamma}{a^2}) \frac{1}{b} , \qquad \gamma = 0.00426 . \tag{44}$$

Here let us try to choose $a = b^{-1}$ as the scale unit, then we obtain

$$\mathcal{M} = \gamma b , \qquad \gamma = 0.00426 , \tag{45}$$

or

$$\tilde{\mathcal{M}} = \gamma, \qquad \gamma = 0.00426 , \tag{46}$$

in the dimensionless expression. because $\tilde{b} = 1$ in this case. This implies that the constant γ is nothing other than the "particle mass", being independent of the *universal length*, on the scale adjusted by $a = b^{-1}$.

Finally, let us examine whether our "particle mass" $\mathcal{M}$ can be regarded as a sort of particle mass in the sense of conventional field theory. For this purpose, we should numerically compute Fourier transform $\tilde{\Delta}_{ii}(k^2)$ given by

$$\tilde{\Delta}_{ii}(k^2) = \int\limits_0^N \frac{dx_0}{\sqrt{N}} \int\limits_0^N \frac{dx_3}{\sqrt{N}} e^{-ik_0 x_0 - ik_3 x_3} \Delta_{ii}(x_3, x_0) , \tag{47}$$

N standing for lattice size. If $\mathcal{M}$ meant a sort of particle mass in this sense, we could hardly observe so sharp $\mathcal{M}$-dependence of $\tilde{\Delta}_{ii}(k^2)$ as a function of k^2, for $\sqrt{k^2} \gg \mathcal{M} = 0.0415, 0.0835, 0.1260, 0.1693, 0.2162$. Figure 4 shows our numerical results (see [4] for technical details), in which all curves for various b's are normalized to $\tilde{\Delta}_{ii}(0) = 1$. We can clearly observe in Fig. 4 that $\tilde{\Delta}_{ii}(k^2)$ is almost independent of k^2, except in its height. That is to say, our anticipation seems justified. For comparison, we put the dashed curve representing a free-propagater (being a substitute of massless free Maxwell field), $\epsilon^2/(k^2 + \epsilon^2)$, with a very small mass $\epsilon = 0.001 \ll \mathcal{M}$.

In order to reconfirm this fact, we present Fig. 5, (for $\frac{1}{\mathcal{M}^2}\Delta_{ii}(k^2)$ versus k^2) stressing that all curves overlap each others for larger k^2. Note that long

Fig. 4. $\tilde{\Delta}_{11}(k^2)$ corresponding to masses obtained in Table 1

Fig. 5. $\frac{1}{\mathcal{M}^2}\tilde{\Delta}_{ii}(k^2)$ versus k^2

distance behavior of the propagater in configuration space gives $\mathcal{M}$, while short distance behavior determines Fourier transforms for larger k^2.

Moreover, we compare a special $\tilde{\Delta}_{ii}(k^2)$ with the corresponding Feynman propagater $\mathcal{M}^2/(k^2 + \mathcal{M}^2)$ for $\mathcal{M} = 0.126$ in Fig. 6, as an example. In this figure one could observe that the difference between them would represent a possible (damping) effect due to the non-linearity of the Born-Infeld field.

Fig. 6. Comparison of $\tilde{\Delta}_{11}(k^2)$ with $\mathcal{M}^2/(k^2 + \mathcal{M}^2)$ for $\mathcal{M} = 0.126$.

5   Conclusion

Summarizing, we have stochastically quantized the Born-Infeld field, characterized by the so-called *universal length*, which cannot be dealt with by means of the conventional quantization methods. Even though we can hardly justify the whole procedure theoretically, we have derived the "particle mass" associated with the (transverse) Born-Infeld field, as a function of the *universal length*, through the conventional formulas to give them.

It would be interesting to observe that we have derived the "particle mass" from a perfectly gauge-invariant field theory. Of course, we can guess that the "particle mass" is produced by introducing the *universal length* b^{-1} having the dimension of length.

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